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INTERACTION-FREE MULTIVALUED DEPENDENCY SETS

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Abstract. *Multivalued dependencies* (MVD) and *join dependencies* were introduced to study database design problems. Join dependencies can be divided into *cyclic* and *acyclic* join dependencies. A fundamental result in dependency theory is that a join dependency is acyclic if and only if it is equivalent to a set of MVDs. In addition to this characterization it has been shown that such a set of MVDs has a *cover* which is a *conflict-free MVD set.* Although this result gives considerable insight into the "syntactic" structure of a set of MVDs equivalent to an acyclic join dependency, the definition of a conflict-free MVD set is complex and does not provide insight into what is meant by the "semantic" notion of a set of MVDs free of conflicts. We introduce the concept of an *interaction-free MVD set.* In contrast to the definition of a conflict-free MVD set, the definition of an interaction-free MVD set clearly indicates that we are dealing with a set of MVDs which do not interact in an adverse way. Furthermore, we provide a simple syntactic characterization of an interaction-free MVD set and show that a join dependency is acyclic if and only if it is equivalent to an interaction-free MVD set.

1. Introduction

Multivalued dependencies [11, 32] (MVD) and *join dependencies* [1, 24] were introduced to study data base design problems. Although join dependencies are more general than MVDs, the latter are easier to work with, both conceptually and technically. A natural question is therefore to study under which conditions and how join dependencies are related to MVDs.

Join dependencies can be divided into two classes: *cyclic* and *acyclic* join dependencies. The latter class contains the MVDs as a proper subclass. One of the fundamental results in dependency theory is the following "semantic" characterization of acyclic join dependencies obtained by Fagin et al. [13]: *A join dependency is acyclic if and only if it is equivalent to a set of MVDs.* In addition to this characterization, Beeri et al. [7] have shown that such a set of MVDs has a *cover* which is a *conflict-free MVD set,* a concept first studied by Lien [20] and Sciore [27]. Although this result gives considerable insight into the "syntactic" structure of a set of MVDs equivalent to an acyclic join dependency, the definition of a conflict-free MVD set is complex and does not provide insight into what is meant by the "semantic" notion of a set of MVDs free of conflicts.

In this paper we introduce the concept of an *interaction-free MVD set.* In contrast to the definition of a conflict-free MVD set, the definition of an interaction-free MVD set clearly indicates that we are dealing with a set of MVDs which do not interact in an adverse way. Furthermore, we provide a simple syntactic characterization of an interaction-free MVD set and show that a join dependency is acyclic if and only if it is equivalent to an interaction-free MVD set.

In Section 2 we review some of the basic definitions in the relational model. In Section 3 we show the relationship between acyclic join dependencies and conflict-free MVD sets. In Section 4 we introduce the concept of an interaction-free MVD set and give a syntactic characterization. In Section 5 we compare the notions of conflict-free and interaction-free MVD sets. The main result of this section is that a join dependency is acyclic if and only if it is equivalent to an interaction-free MVD set.

2. Basic concepts

Let Ω denote the *universe of attributes.* Each attribute $A \in \Omega$ has a set of associated values, its *domain,* dom(A). We will assume that the domain of each attribute has at least two elements. A *relation scheme* R is a finite subset of Ω. A *tuple* t over the relation scheme R is a mapping from R into $\bigcup_{A \in R}(\mathrm{dom}(A))$ such that $t(A) \in \mathrm{dom}(A)$ for each $A \in R$. A *relation* over R is a finite set of tuples over R.

Let t be a tuple over R and let $X \subseteq R$. The *X-value* of t, denoted $t[X]$, is the restriction of the mapping t to X. Let r be a relation over R and $X \subseteq R$; then the *projection* of r on X is the relation $\Pi_X(r) = \{t[X] \mid t \in r\}$. Let $r_1, \ldots, r_n$ be relations over the relation schemes $R_1, \ldots, R_n$ respectively. The join of the relations $r_1, \ldots, r_n$, denoted $r_1 \bowtie \cdots \bowtie r_n$, is the set of tuples t over $\bigcup_{i=1}^{n} R_i$ such that, for each i, $1 \leq i \leq n$, $t[R_i] \in r_i$.

In this paper we will consider two classes of data dependencies, the class of join dependencies and the class of MVDs. Let R be a relation scheme and let $R_1, \ldots, R_n$ be subsets of R such that $R = \bigcup_{i=1}^{n} R_i$. If the relation $r = \Pi_{R_1}(r) \bowtie \cdots \bowtie \Pi_{R_n}(r)$, we say that r *satisfies* the *join dependency* (JD) $\bowtie\{R_1, \ldots, R_n\}$. A *multivalued dependency* (MVD) is a special case of a JD. Let X and Y be disjoint subsets of R. An MVD $X \to\to Y$ for a relation on R is a JD $\bowtie\{XY, XZ\}$, where $Z = R - XY$.

Let $\mathcal{J}_1$ and $\mathcal{J}_2$ be sets of JDs on R. We say that $\mathcal{J}_1$ *logically implies* $\mathcal{J}_2$, denoted $\mathcal{J}_1 \models \mathcal{J}_2$, if and only if whenever a relation r over R satisfies the JDs in $\mathcal{J}_1$, r also satisfies the JDs in $\mathcal{J}_2$. We say that $\mathcal{J}_1$ and $\mathcal{J}_2$ are *logically equivalent,* denoted $\mathcal{J}_1 \equiv \mathcal{J}_2$, if and only if $\mathcal{J}_1 \models \mathcal{J}_2$ and $\mathcal{J}_2 \models \mathcal{J}_1$. If $\mathcal{J}_1 \equiv \mathcal{J}_2$, then $\mathcal{J}_1$ is a *cover* for $\mathcal{J}_2$.

Let r be a relation over R and let $j = \bowtie\{R_1, \ldots, R_n\}$ be a JD on R. We define $\mathrm{CHASE}_j(r)$ as the relation $\Pi_{R_1}(r) \bowtie \cdots \bowtie \Pi_{R_n}(r)$. We can generalize the definition of

CHASE to a set of JDs. Let $\mathcal{J}$ be a set of JDs on R and let r be a relation over R; then $\text{CHASE}_{\mathcal{J}}(r)$ can be defined recursively as follows:

(1) if r satisfies the JDs in $\mathcal{J}$, then $\text{CHASE}_{\mathcal{J}}(r) = r$;

(2) if r violates the JD $j \in \mathcal{J}$, then $\text{CHASE}_{\mathcal{J}}(r) = \text{CHASE}_{\mathcal{J}}(\text{CHASE}_j(r))$.

It was shown by Maier et al. in [21] that $\text{CHASE}_{\mathcal{J}}(r)$ has the finite Church–Rosser property. Furthermore, it follows from the definition that $\text{CHASE}_{\mathcal{J}}(r)$ satisfies the JDs in $\mathcal{J}$.

3. Acyclic join dependencies and conflict-free MVD sets

The class of join dependencies over a relation scheme R contains an important subclass: the acyclic join dependencies. This class can be defined as follows: Let R be a relation scheme and let j be a JD on R; we say that j is an *acyclic* JD if and only if there exists a set of MVDs $\mathcal{M}$ on R such that $\mathcal{M} \equiv \{j\}$. For alternative definitions of acyclic join dependencies and their properties see [2–10, 12–16, 22, 23, 25–29].

A set of MVDs equivalent to an acyclic JD satisfies some interesting properties. In particular, it can be shown that such a set has a cover which is a conflict-free MVD set. The notion of conflict-free MVD sets was introduced by Lien [20], who studied the relationship between the network and relational model. Sciore [27] analysed conflict-free MVD sets in the context of database design and argued that "real-world" sets of MVDs are conflict-free. Sciore showed that a conflict-free MVD set is equivalent to a join dependency. Beeri et al. [7] sharpened this result by showing that a set of MVDs has a conflict-free MVD set cover if and only if it is equivalent to a single (acyclic) JD. More results about conflict-free MVD sets can be found in [8, 17–20, 27, 30].

We use the formalism of [7] to define conflict-free MVD sets. An MVD $X \rightarrow\rightarrow Y$ over the relation scheme R *splits* two attributes A and B if one of them is in Y and the other is in $R - XY$. An MVD *splits* a set V if it splits two attributes in V. A set $\mathcal{M}$ of MVDs over R *splits* a set V if some MVD in $\mathcal{M}$ splits V. We say that a set of $\mathcal{M}$ over R has the *left intersection property* if and only if whenever the MVDs $X \rightarrow\rightarrow Z$ and $Y \rightarrow\rightarrow Z$ are implied by $\mathcal{M}$, then also $X \cap Y \rightarrow\rightarrow Z$ is implied by $\mathcal{M}$. Let $\mathcal{M}$ be a set of MVDs. The left-hand sides of the MVDs of $\mathcal{M}$ are called the *keys* of $\mathcal{M}$. A set $\mathcal{M}$ of MVDs over the relation scheme R is a *conflict-free MVD set* on R if and only if

(1) $\mathcal{M}$ does not split its keys, and

(2) $\mathcal{M}$ has the left intersection property.

Beeri et al. [7] obtained the following important theorem.

Theorem 3.1 (Beeri et al. [7]). *Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. $\mathcal{M}$ has a cover which is a conflict-free MVD set if and only if $\mathcal{M}$ is equivalent to an acyclic join dependency.*

4. Interaction-free MVD sets

The idea of a *conflict-free MVD set* suggests that the MVDs in that set do not "interact in an adverse way". It is not clear, however, what is meant by "interacting in an adverse way". This lack of clarity arises from the rather complicated, nonintuitive syntactic definition of such sets. In this section we propose an alternative definition of MVD sets "free of conflicts". Therefore, we introduce the concept of interaction-free MVD sets.

Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. We say that $\mathcal{M}$ is an *interaction-free MVD set* on R if and only if, for any relation r over R and any pair of MVDs $m_1, m_2 \in \mathcal{M}$,

$$\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r)) = \text{CHASE}_{m_1}(\text{CHASE}_{m_2}(r)).$$

The following example shows that the notion of conflict-free and interaction-free MVD sets are incompatible.

Example 4.1. Goodman and Tay [17] give an example of a conflict-free MVD set which is not an interaction-free MVD set. Let $R = ABCD$ and $\mathcal{M} = \{A \to\to BC \mid D, AB \to\to C \mid D, AC \to\to B \mid D\}$. It can be shown that $\mathcal{M}$ is a conflict-free MVD set, but not an interaction-free MVD set. Indeed, it can be verified that

$$\text{CHASE}_{AB\to\to C|D}(\text{CHASE}_{AC\to\to B|D}(r)) \neq \text{CHASE}_{AC\to\to B|D}(\text{CHASE}_{AB\to\to C|D}(r)),$$

where r is the relation shown in Fig. 1.

Let $R = ABCDE$ and $\mathcal{M} = \{A \to\to BC \mid DE, ABD \to\to C \mid E\}$. It can be verified that $\mathcal{M}$ is an interaction-free MVD set, but not a conflict-free MVD set since the key ABD is split by $A \to\to BC \mid DE$.

A	B	C	D
0	0	0	0
0	0	1	0
0	1	1	1

Fig. 1.

The following theorem gives a characterization of interaction-free MVD sets.

Theorem 4.2. *Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. $\mathcal{M}$ is an interaction-free MVD set on R if and only if*

$$\text{CHASE}_{\mathcal{N}}(r) = \text{CHASE}_{n_p}(\text{CHASE}_{n_{p-1}}(\ldots(\text{CHASE}_{n_1}(r))\ldots))$$

for any relation r on R, any $\mathcal{N} \subseteq \mathcal{M}$, and any sequence of MVDs $(n_1, \ldots, n_p)$ such that

(1) *$p = |\mathcal{N}|$, and*

(2) *$\{n_1, \ldots, n_p\} = \mathcal{N}$.*

Proof. If $|\mathcal{M}| \leq 1$, the theorem is vacuously true. Therefore, assume $|\mathcal{M}| > 1$. The "if" part is trivial, one merely has to consider two-element subsets $\mathcal{N}$ of $\mathcal{M}$. We now show the "only-if" part of the theorem. Let $\mathcal{N} \subseteq \mathcal{M}$ and let $(n_1, \ldots, n_p)$ be a sequence of MVDs such that

(1) $p = |\mathcal{N}|$, and

(2) $\{n_1, \ldots, n_p\} = \mathcal{N}$.

Let $r' = \text{CHASE}_{n_p}(\text{CHASE}_{n_{p-1}}(\ldots(\text{CHASE}_{n_1}(r))\ldots))$. If we can show that r' satisfies the MVDs in $\mathcal{N}$, the result follows immediately from the definition of $\text{CHASE}_{\mathcal{N}}(r)$. Since CHASE_{n_p} was the last CHASE operation, r' satisfies the MVD n_p. Since $\mathcal{N}$ is an interaction-free MVD set, it follows that $r' = \text{CHASE}_{n_{p-1}}(\ldots(\text{CHASE}_{n_1}(\text{CHASE}_{n_p}(r)))\ldots)$, which implies that r' also satisfies the MVD n_{p-1}. This argument can be repeated for the other MVDs in the sequence. Thus r' satisfies the MVDs in $\mathcal{N}$. □

Corollary 4.3. *Let r be a relation over R and let $\mathcal{M}$ be an interaction-free MVD set on R. Then*

$$\text{CHASE}_{\mathcal{M}}(r) = \text{CHASE}_{m_p}(\text{CHASE}_{m_{p-1}}(\ldots(\text{CHASE}_{m_1}(r))\ldots))$$

for any sequence of MVDs $(m_1, \ldots, m_p)$ such that

(1) *$p = |\mathcal{M}|$, and*

(2) *$\{m_1, \ldots, m_p\} = \mathcal{M}$.*

The converse of Corollary 4.3 is not true:

Example 4.4. Consider the MVDs $m_1 = A \to\to BC \mid D$, $m_2 = AB \to\to C \mid D$, and $m_3 = AC \to\to B \mid D$ of Example 4.1, and consider the relation r shown in Fig. 1. It can be verified that, for any permutation (p_1, p_2, p_3) of the MVDs m_1, m_2, and m_3,

$$\text{CHASE}_{\{m_1, m_2, m_3\}}(r) = \text{CHASE}_{p_3}(\text{CHASE}_{p_2}(\text{CHASE}_{p_1}(r))).$$

However, as was shown in Example 4.1, the set of MVDs $\{m_1, m_2, m_3\}$ is not an interaction-free MVD set.

Theorem 4.2 gives a "semantic" characterization of an interaction-free MVD set. A "syntactic" characterization follows in a straightforward way from Theorem 4.5.

Theorem 4.5. *Let R be a relation scheme and let m_1 and m_2 be the MVDs $X \to\to Y \mid Z$ and $U \to\to V \mid W$ on R. Then*

$$\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r)) = \text{CHASE}_{m_1}(\text{CHASE}_{m_2}(r))$$

for any relation r on R if and only if

(1) *$X = U$, or*

(2) $X \cap V = \emptyset$, $Y \cap U = \emptyset$, *and* $Y \cap V = \emptyset$ (*up to renaming of* Y *and* Z, *or* V *and* W[1]), *or*

(3) $m_1 \models m_2$, *or*

(4) $m_2 \models m_1$.[2]

Proof. We first show the "if" direction.

(1) Assume $X = U$. Let r be a relation over R. Because of the symmetry between m_1 and m_2, we only have to show that $\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ satisfies m_1. Let $t_1, t_2 \in \text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ be of the form

	X	$Y \cap V$	$Y \cap W$	$Z \cap V$	$Z \cap W$
$t_1 =$	x	yv	yw	zv	zw
$t_2 =$	x	yv'	yw'	zv'	zw'

We have to show that $\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ contains the tuple t_3 such that

$t_3 =$	x	yv	yw	zv'	zw'

Since $t_1, t_2 \in \text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$, we know that $\text{CHASE}_{m_1}(r)$ contains the tuples t_{11}, t_{12}, t_{21}, and t_{22} of the form:

$t_{11} =$	x	yv	δ_1	zv	δ_2
$t_{12} =$	x	δ_3	yw	δ_4	zw
$t_{21} =$	x	yv'	δ_5	zv'	δ_6
$t_{22} =$	x	δ_7	yw'	δ_8	zw'

Since $\text{CHASE}_{m_1}(r)$ satisfies the MVD $X \to\to Y|Z$, $\text{CHASE}_{m_1}(r)$ also contains the tuples:

x	yv	δ_1	zv'	δ_6
x	δ_3	yw	δ_8	zw'

and therefore $\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ contains the tuple t_3.

(2) Let m_1 and m_2 be the MVDs $X \to\to Y|Z$ and $U \to\to V|W$ such that $X \cap V = \emptyset$, $Y \cap U = \emptyset$, and $Y \cap V = \emptyset$. Because of the symmetry between m_1 and m_2, we only have to show that, for any relation r on R, $\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ satisfies the MVD m_1. Let $t_1, t_2 \in \text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ be of the form:

	$X \cap U$	$X \cap W$	$Y \cap W$	$Z \cap U$	$Z \cap V$	$Z \cap W$
$t_1 =$	xu	xw	yw	zu	zv	zw
$t_2 =$	xu	xw	yw'	zu'	zv'	zw'

We have to show that $\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ contains the tuple t_3 such that

$t_3 =$	xu	xw	yw	zu'	zv'	zw'

[1] Condition (2) can be replaced by either of the following conditions:
- $X \cap V = \emptyset$, $Z \cap U = \emptyset$, and $Z \cap V = \emptyset$, or
- $X \cap W = \emptyset$, $Y \cap U = \emptyset$, and $Y \cap W = \emptyset$, or
- $X \cap W = \emptyset$, $Z \cap U = \emptyset$, and $Z \cap W = \emptyset$.

[2] By [13, Theorem 2], it follows that $m_1 \models m_2$ if and only if (($XY \subseteq UV$ or $XY \subseteq UW$) and ($XZ \subseteq UV$ or $XZ \subseteq UW$)) and $m_2 \models m_1$ if and only if (($UV \subseteq XY$ or $UV \subseteq XZ$) and ($UW \subseteq XY$ or $UW \subseteq XZ$)).

Since $t_1, t_2 \in \text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$, we know that $\text{CHASE}_{m_1}(r)$ contains the tuples t_{11}, t_{12}, t_{21}, and t_{22} such that

$$\begin{array}{lllllll}
t_{11} = & xu & \delta_1 & \delta_2 & zu & zv & \delta_3 \\
t_{12} = & xu & xw & yw & zu & \delta_4 & zw \\
t_{21} = & xu & \delta_5 & \delta_6 & zu' & zv' & \delta_7 \\
t_{22} = & xu & xw & yw' & zu' & \delta_8 & zw'
\end{array}$$

Since $\text{CHASE}_{m_1}(r)$ satisfies the MVD $X \to\to Y$, $\text{CHASE}_{m_1}(r)$ also contains the tuple

$$\begin{array}{llllll} xu & xw & yw & zu' & \delta_8 & zw' \end{array}$$

and therefore $\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ contains the tuple t_3.

(3) Trivial.

(4) Trivial.

We show the "only-if" direction of the theorem by contraposition, i.e., if the MVDs m_1 and m_2 violate conditions (1), (2), (3) and (4), then we show that there exists a relation r over R such that

$$\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r)) \neq \text{CHASE}_{m_1}(\text{CHASE}_{m_2}(r)).$$

In fact, it suffices to show that there exists a relation r such that $\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ violates the MVD m_1. If we assume that $X \to\to Y|Z$ and $U \to\to V|W$ violate conditions (1), (2), (3), and (4), then

$$\begin{aligned}
&X \neq U \quad \text{and}\\
&(X \cap V \neq \emptyset \text{ or } Y \cap U \neq \emptyset \text{ or } Y \cap V \neq \emptyset) \quad \text{and}\\
&(X \cap V \neq \emptyset \text{ or } Z \cap U \neq \emptyset \text{ or } Z \cap V \neq \emptyset) \quad \text{and}\\
&(X \cap W \neq \emptyset \text{ or } Y \cap U \neq \emptyset \text{ or } Y \cap W \neq \emptyset) \quad \text{and}\\
&(X \cap W \neq \emptyset \text{ or } Z \cap U \neq \emptyset \text{ or } Z \cap W \neq \emptyset) \quad \text{and}\\
&((XY \not\subseteq UV \text{ and } XY \not\subseteq UW) \quad \text{or} \quad (XZ \not\subseteq UV \text{ and } XZ \not\subseteq UW)) \quad \text{and}\\
&((UV \not\subseteq XY \text{ and } UV \not\subseteq XZ) \quad \text{or} \quad (UW \not\subseteq XY \text{ and } UW \not\subseteq XZ)).
\end{aligned}$$

It can be verified that the truth assignments, shown in Table 1, (and their consequences[3]) are the only truth assignements to the predicates $X \cap U \neq \emptyset$, $X \cap V \neq \emptyset$, $X \cap W \neq \emptyset$, $Y \cap U \neq \emptyset$, $Y \cap V \neq \emptyset$, $Y \cap W \neq \emptyset$, $Z \cap U \neq \emptyset$, $Z \cap V \neq \emptyset$, and $Z \cap W \neq \emptyset$ which make the above expression true.

By the symmetry of m_1 and m_2, however, there remain only three cases to consider:

(a) Truth assignments (1), (2), (3), and (6) (in Table 1) cannot be distinguished and correspond to the case where $X \cap V \neq \emptyset$, $Y \cap W \neq \emptyset$, and $Z \cap W \neq \emptyset$.

(b) Truth assignments (4), (5), (7), and (8) cannot be distinguished and correspond to the case where $X \cap V \neq \emptyset$, $X \cap W \neq \emptyset$, $Y \cap U \neq \emptyset$ and $Z \cap U \neq \emptyset$.

[3] We say that a truth assignement $(j_{X\cap U}, j_{X\cap V}, j_{X\cap W}, j_{Y\cap U}, j_{Y\cap V}, j_{Y\cap W}, j_{Z\cap U}, j_{Z\cap V}, j_{Z\cap W})$ is a *consequence* of the truth assignment $(i_{X\cap U}, i_{X\cap V}, i_{X\cap W}, i_{Y\cap U}, i_{Y\cap V}, i_{Y\cap W}, i_{Z\cap U}, i_{Z\cap V}, i_{Z\cap W})$ if $j_{X\cap U} \geq i_{X\cap U}, j_{X\cap V} \geq i_{X\cap V}, \ldots, j_{Z\cap W} \geq i_{Z\cap W}$.

Table 1

	$X \cap U \neq \emptyset$	$X \cap V \neq \emptyset$	$X \cap W \neq \emptyset$	$Y \cap U \neq \emptyset$	$Y \cap V \neq \emptyset$
(1)	0	0	0	0	1
(2)	0	0	0	1	0
(3)	0	0	1	0	1
(4)	0	0	1	0	1
(5)	0	0	1	1	0
(6)	0	1	0	0	0
(7)	0	1	0	0	0
(8)	0	1	0	1	0
(9)	0	1	1	1	0

	$Y \cap W \neq \emptyset$	$Z \cap U \neq \emptyset$	$Z \cap V \neq \emptyset$	$Z \cap W = \emptyset$
(1)	1	1	0	0
(2)	0	0	1	1
(3)	0	0	1	0
(4)	0	1	0	0
(5)	0	0	1	0
(6)	1	0	0	1
(7)	1	1	0	0
(8)	0	0	0	1
(9)	0	1	0	0

(c) Truth assignement (9) corresponds to the case $X \cap V \neq \emptyset$, $X \cap W \neq \emptyset$, $Y \cap U \neq \emptyset$, and $Z \cap U \neq \emptyset$.

For each of these cases, we can construct a relation r such that $\text{CHASE}_{m_2}(\text{CHASE}_{m_1}(r))$ violates the MVD $m_1 = X \twoheadrightarrow Y | Z$. The relation shown in Fig. 2, covers case (a). The relation shown in Fig. 3, covers case (b). The relation shown in Fig. 4, covers case (c). Using the examples shown in Figs. 2, 3, and 4, one can construct, in the obvious way, examples for the truth assignments which are either equivalent to or consequences of those mentioned in (a), (b), and (c). □

$X \cap V$	$Y \cap W$	$Z \cap W$
0	0	0
1	1	1

Fig. 2.

$X \cap W$	$Y \cap W$	$Z \cap U$
0	0	0
0	0	1
1	1	1

Fig. 3.

$X \cap V$	$X \cap W$	$Y \cap U$	$Z \cap U$
0	0	0	0
0	1	1	1
1	0	1	1

Fig. 4

5. Fitting it all together

Example 4.1 shows that the notions of conflict-free MVD sets are incompatible. In this section we will show however that they do have a common ground. To do so, we need the notion of a set of MVDs having the subset property, introduced by Goodman and Tay [17].

Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. We say that $\mathcal{M}$ has the *subset property* if and only if for each pair of MVDs $X \to\to Y|Z$ and $U \to\to U|W$ in $\mathcal{M}$,

$$XY \subseteq UV \quad \text{and} \quad UW \subseteq XZ$$

up to renaming of Y and Z, or V and W.[4]

Goodman and Tay obtained the following results.

Lemma 5.1 (Goodman and Tay [17]). *Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. If $\mathcal{M}$ has the subset property, then $\mathcal{M}$ is a conflict-free MVD set.*

The converse of Lemma 5.1 is not true. The set $\mathcal{M}$ of Example 4.1 is a conflict-free MVD set, but the MVDs $AB \to\to C|D$ and $AC \to\to B|D$ violate the subset property. However, the following is true.

Lemma 5.2 (Goodman and Tay [17]). *Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. If $\mathcal{M}$ is a conflict-free MVD set, then $\mathcal{M}$ has a cover which has the subset property.*

Lemmas 5.1, 5.2, and Theorem 3.1 together imply the following theorem.

Theorem 5.3 (Goodman and Tay [17]). *Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. $\mathcal{M}$ has a cover which has the subset property if and only if $\mathcal{M}$ is equivalent to an acyclic join dependency on R.*

In the remainder of this section, we will show that the results similar to Lemmas 5.1, 5.2, and Theorem 5.3 exist between interaction-free MVD set and sets of MVDs which have the subset property.

Lemma 5.4. *Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. If $\mathcal{M}$ has the subset property, then $\mathcal{M}$ is an interaction-free MVD set.*

Proof. Let $X \to\to Y|Z$ and $U \to\to V|W \in \mathcal{M}$. Since $\mathcal{M}$ has the subset property, we may assume, without loss of generality, that $XY \subseteq UW$ and $UV \subseteq XZ$ (cf. footnote[4]).

[4] That is, one of the following is true:
- $XY \subseteq UV$ and $UW \subseteq XZ$,
- $XY \subseteq UW$ and $UV \subseteq XZ$,
- $XZ \subseteq UV$ and $UW \subseteq XY$,
- $XZ \subseteq UW$ and $UV \subseteq XY$.

Condition $XY \subseteq UW$[5] implies that $X \cap V = \emptyset$ and $Y \cap V = \emptyset$. Condition $UV \subseteq XZ$ implies that $Y \cap U = \emptyset$ and $Y \cap V = \emptyset$. Hence $X \to\to Y|Z$ and $U \to\to V|W$ satisfies $X \cap V = \emptyset$, $Y \cap U = \emptyset$, and $Y \cap V = \emptyset$. The result follows from the "if" part of Theorem 4.5. □

Lemma 5.5. *Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. If $\mathcal{M}$ is an interaction-free MVD set, then $\mathcal{M}$ has a cover which has the subset property.*

Proof. Consider the algorithm shown in Fig. 5. It can easily be seen that algorithm transform terminates. We now prove that transform returns a cover of $\mathcal{M}$ which has the subset property. The proof will be by induction on the number of times transform is called before halting.

input: an interaction-free MVD set $\mathcal{M}$
output: a cover of $\mathcal{M}$ which has the subset property

function transform($\mathcal{M}$: set of MVDs): set of MVDs
begin
case
- $\mathcal{M}$ contains a pair of MVDs m_1, m_2 $(m_1 \neq m_2)$ such that $m_1 \models m_2$: **return**(transform($\mathcal{M} - \{m_2\}$))
- $\mathcal{M}$ contains a pair of MVDs $m_1 = X \to\to Y|Z$ and $m_2 = X \to\to V|W$ $(m_1 \neq m_2)$ such that m_1 and m_2 violate the subset property:
  return(transform($\mathcal{M} - \{X \to\to Y|Z, X \to V|W\} \cup \{X \to\to Y \cap V | Y \cap W | Z \cap V | Z \cap W\}$))
- **otherwise**: **return**($\mathcal{M}$)

end(transform).

Fig. 5.

Induction Hypothesis: If $\mathcal{M}$ is an interaction-free MVD set and it takes $k \geq 1$ calls before transform with input $\mathcal{M}$ halts, then transform returns a cover of $\mathcal{M}$ which has the subset property.

Base Step: $k = 1$. In this case, transform returns from the **otherwise** clause. Thus, $\mathcal{M}$ is an interaction-free MVD set which contains no pair of MVDs $m_1 = X \to\to Y|Z$ and $m_2 = U \to\to V|W$ such that:

(a) $m_1 \models m_2$, or
(b) $X = U$ and m_1 and m_2 violate the subset property.

It follows from Theorem 4.5 that, for each pair of MVDs $X \to\to Y|Z$, $U \to\to V|W \in \mathcal{M}$,

$$X \cap V = \emptyset, \quad Y \cap U = \emptyset, \quad \text{and} \quad Y \cap V = \emptyset$$

up to renaming of Y and Z, or V and W. Conditions $X \cap V = \emptyset$ and $Y \cap V = \emptyset$ imply that $XY \subseteq UW$. Conditions $Y \cap U = \emptyset$ and $Y \cap V = \emptyset$ imply that $UV \subseteq XZ$. Thus, $\mathcal{M}$ has the subset property. Since $\mathcal{M}$ is a cover of itself, the induction hypothesis is true in this case.

[5] Remember that X, Y, and Z are pairwise disjoint subsets such that $XYZ = R$. A similar remark is true for the sets U, V, and W.

Induction Step: If transform, with input $\mathcal{M}$, was called $k+1$ times ($k \geq 1$), then the first call must have been a call that resulted from either the first or the second statement in the *case* statement.

Assume the first call was transform $(\mathcal{M}-\{m_2\})$, where $m_2 \in \mathcal{M}$ and such that there exists an $m_1 \in \mathcal{M}$ ($m_1 \neq m_2$) with $m_1 \models m_2$. Clearly, $\mathcal{M}-\{m_2\}$ is a cover of $\mathcal{M}$. Since $\mathcal{M}$ is an interaction-free MVD set, so is $\mathcal{M}-\{m_2\}$. By the induction hypothesis, $\mathcal{M}-\{m_2\}$ has a cover $\mathcal{N}$ which has the subset property. Since $\mathcal{N}$ is a cover for $\mathcal{M}-\{m_2\}$ and since $\mathcal{M}-\{m_2\}$ is a cover of $\mathcal{M}$, the induction hypothesis is true in this case.

Assume the first call was

$$\text{transform}(\mathcal{M}-\{X \to\to Y|Z, X \to\to V|W\} \cup \{X \to\to Y\cap V|Y\cap W|Z\cap V|Z\cap W\}).$$

This implies that for any pair of MVDs m_1 and m_2 in $\mathcal{M}$, $m_1 \not\models m_2$ and $m_2 \not\models m_1$. Let

$$\mathcal{M}' = \mathcal{M}-\{X \to\to Y|Z, X \to\to V|W\} \cup \{X \to\to Y\cap V|Y\cap W|Z\cap V|Z\cap W\}.$$

It follows from the *right intersection* and *union* properties of MVDs [31] that $\mathcal{M}'$ is a cover for $\mathcal{M}$. It can also be verified, by using Theorem 4.5, that $\mathcal{M}'$ is an interaction-free MVD set (remember $\mathcal{M}$ is an interaction-free MVD set). The induction hypothesis implies that $\mathcal{M}'$ has a cover $\mathcal{N}$ which has the subset property. Since $\mathcal{M}'$ is a cover of $\mathcal{M}$, $\mathcal{N}$ is also a cover for $\mathcal{M}$ and the induction hypothesis is true. □

Lemmas 5.4, 5.5, and Theorem 5.3 together imply the following theorem.

Theorem 5.6. *Let R be a relation scheme and let $\mathcal{M}$ be a set of MVDs on R. $\mathcal{M}$ has a cover which is an interaction-free MVD set if and only if $\mathcal{M}$ is equivalent to an acyclic join dependency on R.*

We can now state the main result of this section.

Theorem 5.7. *Let R be a relation scheme. A join dependency is acyclic if and only if it is equivalent to an interaction-free MVD set.*

Proof. Follows immediately from Theorem 5.6 and the definition of a cover. □

6. Conclusion

Theorem 5.7 adds another fact to the arsenal of conditions that characterize acyclic join dependencies. In comparison to the other conditions that relate acyclic join dependencies to MVDs, we feel that the concept of interaction-free MVD sets has the advantage of providing a simple semantic characterization.

We conclude by mentioning an open problem: Is there a simple characterization for a set of MVDs which satisfies the conditions specified in Corollary 4.3?

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